Adjustment capacity in a monetary union: a DSGE evaluation of Poland and Slovakia

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VERY PRELIMINARY!

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Abstract

Efficient functioning of the competitiveness channel is broadly considered the main substitute for the abandoned autonomous monetary and exchange rate policy once a country joins a monetary union. This paper attempts to make an empirical assessment of how the real exchange rate – being a basic measure of price competitiveness of domestic producers abroad – stabilizes the Polish economy in comparison to the real interest rate. To address this issue, we revisit the concept of weighting the Monetary Conditions Index (MCI-ratio) to explore the relative importance of the two channels. We apply the IS-curve approach, building on a small open economy DSGE framework with forward-looking estimation (GMM and FIML). We conclude that the dominance of competitiveness channel over the real interest rate effect in Poland can be remarkable. We compare the estimates for Poland to Slovakia, concluding that the latter country seems to be more capable of handling asymmetric shocks under the common monetary policy. Finally, in the context of Slovak revaluations in ERM II, we consider the choice of central parity as a policy tool in the face of natural interest rate differential against the euro area and examine its efficiency given the estimated MCI-ratios.

Keywords: competitiveness channel, EMU, MCI-ratio, DSGE, forward-looking estimation.

JEL Classification: C22, C32, F15, F41.

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1 Introduction

Once a country joins a monetary union, its capacity of absorbing asymmetric shocks via policy tools is significantly reduced. Namely, it abandons two main macroeconomic degrees of freedom: autonomous monetary policy and, to a large extent, nominal exchange rate volatility. The burden of adjustment shifts to a large extent to market-based mechanisms, described in the literature as the competitiveness channel (de Grauwe, 2007; European Commission, 2008). After an asymmetric shock and in the absence of nominal appreciation or depreciation, as well as fine-tuning possibilities with country-specific nominal interest rate, the speed of reversal to equilibrium is mainly conditional on the price dynamics, and thus on product and labour market flexibility.

In addition, this reversal could be hampered by procyclicality of real interest rates in a monetary union. When a positive demand shock raises the inflation rate and there is a close link between the inflation rate and inflation expectations on the country level, the real interest rate declines, which additionally fuels the boom. A positive impulse for the economic activity can also stem from the fact that an economy with higher natural interest rate and long-term inflation joins a monetary union.

There are dynamic interactions between the competitiveness channel and the real interest rate effect. The former is commonly believed to be an equilibrating force in the long run, whereby the latter – to boost output and inflation volatility via short run effects (Arnold and Kool, 2004; Roubini et al., 2007; European Commission, 2008; Wójcik, 2008). However, a joint consideration of both mechanisms seems to be the appropriate approach to model the adjustment process in the aftermath of an asymmetric shock (see e.g. Hoeller et al., 2004; European Commission, 2006; Torój, 2009a).

This paper aims to contribute to the literature by performing a tentative *ex ante* assessment of the adjustment capacity of Poland by exploring the relative sensitivity of Polish economic activity to the real interest rate and real exchange rate. We also take a comparative perspective and perform analogous estimations for Slovakia – another new EU member state that adopted the euro in 2009. The Slovak experience is of particular interest because of its ERM II experiences. In the course of ERM II participation, the Slovak Koruna was twice revalued, which definitely affected the competitiveness of Slovak goods on international markets.

The question about the relative impact of the real exchange and interest rates on economic activity in a small open economy was heavily discussed around the turn of the centuries, when a concept of Monetary Conditions Index (MCI) was popular among policymakers. The main purpose of the MCI was to weight the interest rate together with the exchange rate to provide a more adequate assessment of monetary policy stance for a small economy than with the interest rate only (see Figure 1 for the estimates for Poland). Out of a number of econometric techniques developed for the purpose of MCI-ratio estimation (i.e. the impact of real interest rate divided by the impact of real exchange rate), we adopt here the IS-curve approach, i.e., we look at the relative importance of both variables in controlling the output gap dynamics. Our dynamic IS curve is part of a micro-founded New Keynesian DSGE model.
With the MCI-ratios for Poland and Slovakia in hand (estimated via GMM and FIML in a rational expectations framework), we perform simulations of the adjustment path after a potentially asymmetric shocks in the euro area. Also, we argue that these ratios could provide some information on the effects of central parity revaluations within the ERM II as a tool that could potentially offset the positive demand pressure from natural interest rate differential between the new member states and Poland.

The rest of the paper is organized as follows. Section 2 reviews the literature on estimating the relative impact of the real interest rate and the real exchange rate on the economic activity (MCI-ratio). In Section 3, a DSGE model is developed. Section 4 contains the estimation results for Poland and Slovakia. In Section 5, the impact of MCI-ratio on the adjustment capacity after an asymmetric shock is simulated. Section 6 investigates the implications of central parity revaluation within the model. Section 7 concludes.

2 MCI-ratio estimation: review of the literature

The relative impact of real interest rate effect and competitiveness channel on small open economies mainly depends on the relative effects of the real interest rate and the real exchange rate shifts for aggregate output and inflation. This impact was investigated for many countries in the context of monetary policy conduct. In order to account for both demand-side factors, central bankers constructed various aggregate measures, including the Monetary Conditions Index (MCI). The ratio of interest rate and exchange rate impact on economic activity is described in the literature as the MCI-ratio. It it is less than one, it suggests that the impact of the real exchange rate is stronger compared to the impact of real interest rate.

Early research was focused on providing a holistic definition of monetary conditions: Fischer and Orr
(1994) use survey data from entrepreneurs who assessed tightness of current monetary policy on a scale of 1 to 7. Although the question was posed in absolute terms, some relationship between the share of foreign market in the own business and the variance of their evaluations was detected, as the same interest or exchange rate developments weighted differently on business agents’ perspectives. Hens (1992) attempts to evaluate monetary policy stance by means of a simple indicator composed of real interest and exchange rates plus yield curve slope and money supply. He sets weights for particular components fully arbitrarily.

The MCI is a weighted average of real interest rate and real exchange rate of a country:

\[ MCI_t = \alpha (r_t - \bar{r}_0) + (1 - \alpha) (rer_t - \bar{rer}_0) \]

with \( \alpha \) denoting relative weight of real interest rate \( (r_t) \) and \( 1 - \alpha \) denoting relative weight of real exchange rate \( (rer_t) \). 0 is the basis period.\(^1\) It has been conceived in early 1990s in the Bank of Canada as a univariate measure of how ‘tight’ or ‘loose’ monetary conditions in an economy are (Freedman, 1994, 1995). The author motivated the index with the necessity to take into account not only interest rate, but also exchange rate developments in a small open economy when measuring monetary policy stance.\(^2\) As Kozłowski (2004) reminds, this was postulated after the collapse of Bretton Woods system. The specification assumes implicitly that both channels are perfectly substitutable and work independently.\(^3\) Mayes and Viren (2002) discusses this substitution on the aggregate level as a synthesis of contradictory reactions at sectoral levels (for example, importers versus exporters). Hyder and Khan (2006) see here space for significant sectoral shifts.

\(^1\)MCI equals zero in the first period; positive or negative value means that monetary conditions are tighter or looser than in the basis period, but not tight or loose in absolute terms, whatever it should mean. Interpretation should be focused on dynamics between any two periods or trends.

\(^2\)The specification above has been generalized in a number of publications to Financial Conditions Index (FCI):

\[ FCI_t = \sum_{k=1}^{K} w_k (P_{k,t} - \bar{P}_{k,t}) \]

with \( P_{k,t} \) denoting price of financial asset \( k \) at time \( t \) (\( P_{k,t} \) long-term trend or equilibrium value thereof) and \( w_k \) relative weight of asset \( k \) in the index. This specification encompasses MCI with real interest and exchange rate as specific financial assets. Gauthier et al. (2004) argue that this extension is relevant for theoretical completeness and stable, significant dynamic correlations with output gap. Rising asset prices should increase economic agents’ perception of own wealth and hence consumption (Grande, 1997), while growing value of firms’ assets provides them with better collateral and improves their credit capacity (balance sheet effect within credit channel, see Mayes and Viren (2001) for a detailed discussion). According to Goodhart and Hofmann (2001), aggregate demand is influenced by all asset prices and we should not skip any of them. The authors suggest housing prices, stock prices and bond risk premium. However, such a specification is problematic in practice: the choice of variable set is arbitrary, they are usually not included in models and the relationships are too vague to be able to deliver a robust specification; relevant data sets normally suffer from strong multicollinearity. For these reasons, and in line with European Commission (2006) considering the wealth effect captured within the real interest rate, we do not follow the FCI approach in this paper.

\(^3\)Ericsson et al. (1998) interpret this class of indices as a three-level aggregation: first, we select broad macroeconomic and financial categories which are expected to represent monetary or financial conditions - say, real interest rate and real exchange rate. Then we select variables to operationalize them, e.g. 3-month money market rate or long term rate, CPI-or ULC-based REER. On the third stage, we search for weights to build a single index. This paper is focused mainly on the third stage.

monetary policymakers in late 1990s and early 2000s, the concept of using the indicator in monetary policy conduct has been given up. However, the indicator can still be a useful source of macroeconomic knowledge, in the spirit of point 3 by Batini and Turnbull (2002), making econometric worries associated with the construction of the index worth resolving.

In order to assess the relative impact of the real interest and exchange rate, a target variable needs to be defined. Eika et al. (1996) consider then the weights as multipliers of the target variable with respect to both rates in question. This nomenclature is rooted in a hierarchy of monetary policy concepts: Batini and Turnbull (2002) present this issue in the light of legally defined scope of central bank actions. As we consider demand-driven impact of the two rates on economic activity, output gap seems to be the most appropriate choice for the target variable. In the literature related to the subject, output gap is usually measured as a percentage deviation of seasonally adjusted real GDP from its value smoothed by modified Hodrick-Prescott filter ($\lambda = 1600$, which is standard for quarterly data).

Much attention used to be paid to the role of MCI in monetary policy conduct. Freedman (1994, 1995) understood it as an indicator, assisting the central bank in tracing the transmission process from policy instruments, over operating targets and intermediate targets to ultimate target. Bofinger (2001) builds a model where MCI helps to minimize the loss function of the central bank subject to internal and external equilibrium conditions, which yields a unique solution for real interest and exchange rate. Stevens (1998) considers MCI as a hybrid of instrument and target of a central bank, as in a free floating regime no direct control exists over the exchange rate. Mayes and Viren (2000) stress in this context the importance of profound knowledge of empirical interactions between the interest and exchange rate. According to Frochen (1996), MCI cannot be treated as a synthetic indicator of monetary policy actions because it takes market-based variables into account; market is also pricing in its own expectations and perception. This point of view seems to have been widely accepted in subsequent literature. Siklos (2000) emphasizes in this context the role of MCI in communication, as a feedback from the financial markets towards the central bank.

Probably the most spectacular pitfalls in MCI-based monetary policy conduct were recorded in New Zealand. As reported by Drew (2001); Mishkin and Schmidt-Hebbel (2007); Svensson (2001), currency depreciation associated with Asian crisis in 1997 tempted Reserve Bank of New Zealand to raise interest rates in order to maintain the MCI at the level assumed earlier. This was accompanied by a drought in the summer 1997/1998 with contractionary effects for internal demand. In the aftermath of the crisis, external demand was too weak to support the expansionary effect of currency depreciation, which resulted in temporary unnecessary swings in output.

Freedman (1994) did not place the problem of weighting and MCI-ratio in the central point of the analysis. He assigns a weight of $\frac{1}{4}$ to the exchange rate so as to reflect the impact of external developments on Canadian economy. In the subsequent literature, more formal methods are investigated. Some authors use an observable proxy for the degree of openness (e.g. share of imports in GDP, as World Bank (2005, 2006) does). Such treatments are subject to criticism for the following reasons: they are decoupled from full complexity of impact of both channels, the choice of time period is arbitrary (the

\footnote{However, this method is recently facing heavy criticism with regard to an application to nonstationary data (see e.g. Lada, 2007).}
World Bank uses e.g. average share over 2000-2005) and we are unable to calculate an index with more sophisticated dynamic specification. This is why the econometric approach is dominating in the literature.

A battery of literature considers inflation as another target variable. Both approaches are similar, as it is the output gap that generates inflationary pressure. However, the pass-through effect of exchange rate normally leads to a direct impact of currency depreciation (appreciation) on rising (falling) prices via import prices, independently on the output gap effects. In consequence, when choosing inflation rate as target variable, higher weight for the exchange rate should be expected than with output gap in the target variable role. A practical advantage of this approach is that it allows us to use monthly data instead of quarterly, which improves statistical properties of the estimation. A serious drawback of this approach is pointed to by Stevens (1998): according to him, treating prices on the aggregate level too general as they are determined in a different way in the tradeable sector (exchange rate pass-through, world market prices) and non-tradeable one (output gap, inflation expectations). This point of view is shared by Mayes and Viren (2000), as inflation requires much more sophisticated econometric treatment in their view. In his pioneering work, Freedman (1994) opts for an IS equation as any pass-through effect are relatively small and irrelevant for Canadian business cycle analysis; arguments of Frochen (1996) remain in line. Kokoszczyński (2004) in turn opts for weights reflecting the entire impact of real interest and exchange rate on prices, not only via output gap.

While the debate over using MCI as monetary policy rule or operating target has reached an un-favourable conclusion for the index, econometric question of estimating the best MCI-ratio for an economy remained unresolved. In the discussion, a few problems have been particularly highlighted:

**Dynamic specification.** A model whose parameters are used to derive weights for MCI components must reflect short- and long-term dynamic properties of the underlying process. The dynamic order of multipliers that we use depends on the desired dynamic interpretation of MCI. Batini and Turnbull (2002) suggests relaxing any arbitrary assumptions as to the horizon of monetary conditions’ impact on the real economy and specifying a model with distributed lags. The parameters for lagged values could be used then in a modified index - Dynamic MCI (DMCI), including not only current, but also properly weighted past values of interest and exchange rates.

**Parameter stability.** Whether model parameters from which multipliers are derived can be treated as stable or not is crucial for the validity of calculations. Among potential reasons for violation of this assumption, changing share of firms’ credit in foreign currency (Deutsche Bundesbank, 1999), structural transformation, monetary integration (Mayes and Viren, 2000) and liquidity problems (Gerlach and Smets, 2000) are the most popular in theoretical literature. Elka et al. (1996) suggest particular vulnerability of reduced-form models to violation of this assumption and criticise the assumption of superexogeneity of interest and exchange rates which is in fact associated with constant weights.

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5Elka et al. (1996) see here a dynamic link between a monetary policy instrument and its ultimate target.

6It must be stressed here that the MCI framework considered here is too simple to fulfill a postulate of completeness; Ericsson et al. (1998) suggests admitting that MCI is reflecting only part of the complex reality and should be interpreted as no more than that.
Nonstationarity. Eika et al. (1996) warns from using too few variables when building cointegrating relationships with I(1) variables, which limits the number of relationships themselves. They recommend a strategy of testing endogeneity and then excluding an equation from VAR, shifting a variable into exogenous variable set should the null not be rejected. Estimation of weights for Poland could additionally be distorted by economic transformation, which implies systematic appreciation of equilibrium REER and nonstationarity, and Balassa-Samuelson effect. With labour productivity and wages growing faster in tradeable sector, which is enforced by international competition, and wages equal across sectors due to local wage arbitrage, the nontradeable sector faces excessive wage growth with respect to productivity and generates inflation in the entire economy. This results in real exchange rate appreciation faster than equilibrium value and tightening of monetary conditions.

Exogeneity. We must not overlook the fact that the map of potential variable interdependencies presented before is incomplete. According to theoretical literature, real interest rate and real exchange rate are also linked by a relationship called uncovered interest parity (UIP): with inflation rate as a policy instrument and free-floating exchange rate, free capital flows should turn positive interest rate disparity with the rest of the world into appreciation with expectations of future depreciation. However, as noted by Frochen (1996); Chwiejczak (1999), this condition seldom holds in the short term. Nonetheless, we must bear it in mind as a potential source of multicollinearity and endogeneity problem. Gottschalk (2001) suggests that monetary conditions cannot be treated as exogenous, at least not in mid- to long term, as central bank is reacting to macroeconomic developments with its policy rate and exchange rate is part of equilibrating mechanism. According to Eika et al. (1996), rejection of hypothesis of interest or exchange rate’s weak exogeneity leads to necessity of model expansion and incorrectness of indirect multipliers.7

MCI-ratio precision. Although both parameters in question were estimated at very different levels across countries, their relative absolute value (which is the focus of this analysis) labelled in the literature as MCI-ratio (by custom, with parameter for the interest rate in numerator and for the exchange rate in denominator) exhibits much less dispersion. Ericsson et al. (1998) calculate confidence intervals for these ratios, using likelihood ratio-based method by Silvey (1978).8 For the models analyzed, they find extremely wide MCI-ratio confidence intervals, including all positive real numbers for Norway and all real numbers for the USA.

Last but not least, a correct identification of sources of shocks also matters for the interpretation of MCI dynamics, evolving automatically in line with real exchange and interest rate trajectories. However, it is more often than not impossible in real time. According to Kokoszczyński (2004); Stevens (1998); Freedman (1994)

- cost-push shocks (e.g. in oil prices) or change in portfolio preferences of domestic investors are generally not expected to influence domestic monetary policy actions;

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7Impact and long-term multipliers remain correct.
8Gregory and Veall (1985) present evidence of this method’s superiority over earlier approaches by Wald (1943); Fieller (1954).
- foreign inflation or policy interest rates and domestic terms-of-trade changes require careful consideration;
- exchange rate shocks associated with lack of the market’s confidence in central bank’s credibility need to be offset.\(^9\)

The most sophisticated econometric approach is to use a complex macroeconomic model in order to obtain dynamic multipliers in response to unit disruption in real interest and exchange rates. However, it is often impossible for practical reasons: models contain these variables in nominal terms or there is a monetary rule that levels off part of the influence immediately (Kot, 2003). Examples of empirical works following this strategy are Kennedy and Riet (1995); Mayes and Viren (2000) and Kokoszczyński (2004). Such a holistic view might also be inferior to more robust specifications due to data limitations and short time series at our disposal.

In this paper, we consider the IS curve approach, treating the output gap as the target variable. The specification of this equation is based upon a small open economy dynamic stochastic general equilibrium model. For the system estimation, as well as for the simulations, we employ the full log-linearized model.

### 3 DSGE model: a 'micro-founded' MCI ratio

The currency union model consists of 2 regions. In further analyses, one can calibrate or estimate the model for any region pair, focusing on the country of interest. The whole economy represented by the model, in line with a conventional treatment in the DSGE literature, \(^10\), is represented by the interval \([0;1]\), whereby the first region (say, home economy) is indexed over \([0;w]\) (relative size of the region: \(w\)), and the second (foreign economy) is indexed over \([w;1]\).

Parameters of the foreign economy are denoted analogously to home economy and marked with a star, e.g. \(\sigma\) and \(\sigma^*\). Both economies consist of two sectors: tradables and nontradables.

#### 3.1 Household decisions

**3.1.1 Intratemporal allocation of consumption**

Households get utility from consumption and disutility from hours worked. In addition, utility from consumption depends on consumption habits formed in the previous period (see Smets and Wouters, 2003; Kolasa, 2008). The constant relative returns to scale utility function takes the form (see Galí, 2008):

\[
U_t (C_t, N_t, H_t) = \frac{(C_t - H_t)^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\phi}}{1 + \phi}
\]

\(^9\) Gottschalk (2001) postulates differentiating between monetary policy shocks, for which MCI would be relevant analysis tools, from supply and demand shocks, when MCI becomes endogenous.

\(^10\) See Benigno (2004); Blessing (2008); Kolasa (2008).
where \( C_t \) — consumption at \( t \), \( H_t \) — stock of consumption habits at \( t \), \( N_t \) — hours worked at \( t \). Consumption habits are proportional to consumption at \( t - 1 \) (see Smets and Wouters, 2003):

\[
H_t = hC_{t-1}
\]

The overall consumption index is aggregated from the tradable and non tradable consumption bundle:

\[
C_t \equiv \left[ (1 - \kappa) \frac{1}{\eta} \frac{\delta_{C_{T,t}^n}}{\eta_{C_{T,t}^n}} + \kappa \frac{1}{\eta} \frac{\delta_{C_{N,t}^n}}{\eta_{C_{N,t}^n}} \right] \frac{\eta_{C_{T,t}^n}}{\eta_{C_{N,t}^n}}
\]

where \( \kappa \in [0; 1] \) characterizes the share of non tradables in the home economy and \( \delta \) is elasticity of consumption substitution between the goods produced in both sectors.

The domestic consumption of tradables at \( t \) consists of goods produced at home, \( C_{H,t} \), and abroad, \( C_{F,t}^* \):

\[
C_{T,t} \equiv \left[ (1 - \alpha) \frac{1}{\eta} \frac{\delta_{C_{H,t}^n}}{\eta_{C_{H,t}^n}} + \alpha \frac{1}{\eta} \frac{\delta_{C_{F,t}^n}}{\eta_{C_{F,t}^n}} \right] \frac{\eta_{C_{H,t}^n}}{\eta_{C_{F,t}^n}}
\]

An analogous relationship holds for the foreign economy. Given this, \( \alpha \) is an intuitive measure of degree of openness and \( 1 - \alpha \) — home bias in consumption. \( \eta > 0 \) is the elasticity of consumption substitution between home and foreign tradables.

A single type of good is indexed as \( k \) and belongs to good variety indexed over the interval \([0; 1]\).

The consumption of domestic tradable goods in the home economy \( (C_{H,t}) \) and in the foreign one \( (C_{H,t}^*) \) is defined as:

\[
C_{H,t} \equiv \left[ \left( \frac{1}{\eta} \right) \frac{1}{\varepsilon_{T}} \int_0^1 \left( \int_0^w C_{j, H, t, k} dj \frac{\varepsilon_{T-1}}{\varepsilon_{T}} \right) dk \right] \frac{\eta_{C_{H,t}^n}}{\eta_{C_{H,t}^n}}
\]

\[
C_{H,t}^* \equiv \left[ \left( \frac{1}{\eta} \right) \frac{1}{\varepsilon_{T}} \int_0^1 \left( \int_0^w C_{j, H, t, k}^* \frac{\varepsilon_{T-1}}{\varepsilon_{T}} \right) dk \right] \frac{\eta_{C_{H,t}^n}}{\eta_{C_{H,t}^n}}
\]

The parameter \( \varepsilon_{T} > 1 \) measures the elasticity of substitution between various types of goods in international trade, \( k \) indexes the variety of goods, and \( j \) — the households (integral over \( j \) reflects the difference in both economies’ size).

We define in an analogous way domestic and foreign consumption of the goods produced abroad, \( C_{F,t} \) and \( C_{F,t}^* \):

\[
C_{F,t} \equiv \left[ \left( \frac{1}{\eta} \right) \frac{1}{\varepsilon_{T}} \int_0^1 \left( \int_0^w C_{j, F, t, k} dj \frac{\varepsilon_{T-1}}{\varepsilon_{T}} \right) dk \right] \frac{\eta_{C_{F,t}^n}}{\eta_{C_{F,t}^n}}
\]

\[
C_{F,t}^* \equiv \left[ \left( \frac{1}{\eta} \right) \frac{1}{\varepsilon_{T}} \int_0^1 \left( \int_0^w C_{j, F, t, k}^* \frac{\varepsilon_{T-1}}{\varepsilon_{T}} \right) dk \right] \frac{\eta_{C_{F,t}^n}}{\eta_{C_{F,t}^n}}
\]
For both tradable consumption baskets, we define equal elasticity of substitution between various types of goods, $\varepsilon^T$, both at home and abroad.

The nontradable consumption bundles, domestic ($C_{N,t}$) and foreign ($C_{N*,t}$), are characterized as:

$$C_{N,t} \equiv \left[ \frac{1}{w} \int_0^1 \left( \int_0^w C_{H,t,k} \, dj \right)^{\frac{\varepsilon_{N-1}}{\varepsilon_N}} \, dk \right] \quad C_{N*,t} \equiv \left[ \frac{1}{1-w} \int_0^1 \left( \int_w^1 C_{N*,t,k} \, dj \right)^{\frac{\varepsilon_{N*,-1}}{\varepsilon_{N*}}} \, dk \right]$$

Households maximize at $t$ the discounted flow of future utilities:

$$E_t \sum_{t}^\infty \beta^t U(C_t, N_t, H_t) \rightarrow \max_{C, N}$$

where $\beta \in (0, 1)$ is households’ discount factor. Maximization of (9) is subject to a sequence of current and future budget constraints:

$$\int_0^1 P_{H,t,k} C_{H,t,k} + \int_0^1 \int_0^1 P_{j,t,k} C_{j,t,k} \, dj \, dk + E_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t \quad \forall_t$$

The right-hand side is a household’s budget at $t$. Its income consists of payoffs of securities acquired in the previous periods ($D_t$), labour incomes ($W_t$ - nominal wage at $t$) and government transfers ($T_t$). The left-hand side of the inequality sums the consumption spendings of households (where $P$ denotes a price of a particular consumption bundle, indexed in line with these bundles) and acquisition of securities. $Q_{t,t+1}$ is a stochastic discount factor for the payoffs at $t+1$, faced by the households.

Maximizing (9) subject to (10) leads to the following first order conditions:

- demand equations (home and foreign) for individual goods $k$ produced at home:

$$C_{H,t,k} = \frac{1}{w} \left( \frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} C_{H,t} \quad C_{H,t,k}^* = \frac{1}{w} \left( \frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} C_{H,t}^*$$

- demand equations (home and foreign) for individual goods $k$ produced abroad:

$$C_{F,t,k} = \frac{1}{1-w} \left( \frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} C_{F,t} \quad C_{F,t,k}^* = \frac{1}{1-w} \left( \frac{P_{F,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} C_{F,t}^*$$

- demand equations (home and foreign) for individual nontradable goods:

$$C_{N,k} = \frac{1}{w} \left( \frac{P_{N,k}}{P_N} \right)^{-\varepsilon_N} C_N \quad C_{N,k}^* = \frac{1}{1-w} \left( \frac{P_{N,k}}{P_N} \right)^{-\varepsilon_{N*}} C_N^*$$

- demand equations (home and foreign) for domestic tradable goods:
\[ C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-\eta} C_{T,t} \]
\[ C_{H,t}^* = \alpha^* \left( \frac{P_{H,t}}{P_{T,t}^*} \right)^{-\eta^*} C_{T,t}^* \quad (14) \]

- demand equations (home and foreign) for foreign tradable goods:
\[ C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_{T,t}} \right)^{-\eta} C_{T,t} \]
\[ C_{F,t}^* = (1 - \alpha^*) \left( \frac{P_{F,t}}{P_{T,t}^*} \right)^{-\eta^*} C_{T,t}^* \quad (15) \]

- home and foreign demand equations for all tradable goods:
\[ C_{T,t} = (1 - \kappa) \left( \frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \]
\[ C_{T,t}^* = (1 - \kappa^*) \left( \frac{P_{T,t}^*}{P_t^*} \right)^{-\delta^*} C_t^* \quad (16) \]

- home and foreign demand equations for all nontradable goods:
\[ C_{N,t} = \kappa \left( \frac{P_{N,t}}{P_t} \right)^{-\delta} C_t \]
\[ C_{N,t}^* = \kappa^* \left( \frac{P_{N,t}^*}{P_t^*} \right)^{-\delta^*} C_t^* \quad (17) \]

- home and foreign labour supply equations:
\[ C_t^\sigma N_t^\sigma = \frac{W_t}{P_t} \]
\[ C_t^\sigma^\sigma N_t^\sigma^\sigma = \frac{W_t^*}{P_t^*} \quad (18) \]

Individual price indices are defined in the following way:

\[ P_{H,t} = \left[ \frac{1}{w} \int_0^1 \left( \int_0 w P_{H,t,k} dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}} \]
\[ P_{H,t}^* = \left[ \frac{1}{1-w} \int_0^1 \left( \int_0 w P_{H,t,k} dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}} \quad (19) \]

\[ P_{F,t} = \left[ \frac{1}{w} \int_0^1 \left( \int_0 w P_{F,t,k} dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}} \]
\[ P_{F,t}^* = \left[ \frac{1}{1-w} \int_0^1 \left( \int_0 w P_{F,t,k} dj \right)^{1-\varepsilon_T} dk \right]^{\frac{1}{1-\varepsilon_T}} \quad (20) \]

\[ P_{T,t} = \left[ (1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \]
\[ P_{T,t}^* = \left[ (1 - \alpha^*) P_{F,t}^{1-\eta} + \alpha^* P_{H,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (21) \]

\[ P_{N,t} = \left( \frac{1}{w} \int_0^1 \left( \int_0 w P_{N,t,k} dj \right)^{1-\varepsilon_N} dk \right)^{\frac{1}{1-\varepsilon_N}} \]
\[ P_{N,t}^* = \left( \frac{1}{1-w} \int_0^1 \left( \int_0 w P_{N,t,k} dj \right)^{1-\varepsilon_N} dk \right)^{\frac{1}{1-\varepsilon_N}} \quad (22) \]

\[ P_t = \left[ (1 - \kappa) P_{T,t}^{1-\delta} + \kappa P_{N,t}^{1-\delta} \right]^{\frac{1}{1-\delta}} \]
\[ P_t^* = \left[ (1 - \kappa^*) P_{N,t}^{1-\delta^*} + \kappa^* P_{N,t}^{1-\delta^*} \right]^{\frac{1}{1-\delta^*}} \quad (23) \]
Log-linearization and differencing the formulas (21) and (23) lead to the following dependencies:

\[ \pi_{T,t} = (1 - \alpha) \pi_{H,t} + \alpha \pi_{F,t} \]
\[ \pi_{T,t}^* = (1 - \alpha^*) \pi_{F,t} + \alpha^* \pi_{H,t} \]

(24)

\[ \pi_t = (1 - \kappa) \pi_{T,t} + \kappa \pi_{N,t} \]
\[ \pi_t^* = (1 - \kappa^*) \pi_{T,t}^* + \alpha \pi_{N,t}^* \]

(25)

Using the above equations, we derive domestic demand functions for the domestic tradable, foreign tradable and non tradable goods:

\[ C_{H,t,k} = \frac{1}{w} (1 - \alpha) (1 - \kappa) \left( \frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} \left( \frac{P_{T,t}}{P_t} \right)^{-\eta} \left( \frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \]

(26)

\[ C_{F,t,k} = \frac{1}{1 - w} \alpha (1 - \kappa) \left( \frac{P_{H,t,k}}{P_{F,t}} \right)^{-\varepsilon_T} \left( \frac{P_{F,t}}{P_{F,t}} \right)^{-\gamma} \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} \left( \frac{P_{T,t}}{P_t} \right)^{-\delta} C_t \]

(27)

\[ C_{N,t,k} = \frac{1}{w} \kappa \left( \frac{P_{N,t,k}}{P_N} \right)^{-\varepsilon_N} \left( \frac{P_{N,t}}{P_t} \right)^{-\delta} C_t \]

(28)

Analogous equations hold for the foreign economy.

3.1.2 Intertemporal allocation of consumption

We define the stochastic discount factor as:

\[ Q_{t,t+1} = \frac{V_{t,t+1}}{\xi_{t,t+1}} \]

(29)

where \( V_{t,t+1} \) is the price at \( t \) of an Arrow security, i.e. a one-period security paying 1 at \( t + 1 \) when a specific state of nature occurs and 0 otherwise. \( \xi_{t,t+1} \) is the probability that the state of nature in which 1 is paid materializes, conditional on the state of nature at \( t \). Having the access to such a security market, households can transfer utility between periods, maximizing its discounted flow (see Galí and Monacelli, 2005).

The optimality of decisions requires that the marginal loss in utility due to buying the security at \( t \) instead of allocating this money to consumption must equal the discounted payoff at \( t + 1 \), also expressed in terms of marginal growth of future utility:

\[ \frac{V_{t,t+1}}{P_t} (C_t - H_t)^{-\sigma} = \xi_{t,t+1} \beta (C_{t+1} - H_{t+1})^{-\sigma} \frac{1}{P_{t+1}} \]

(30)

whereby \( C_{t+1} \) and \( P_{t+1} \) in the above equation should be interpreted as conditional expected values given the state of nature when the payoff is nonzero.
Applying the definition of $Q_{t,t+1}$ (29) and (4), the equation (30) can be written as:

$$\beta \left( \frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}$$ (31)

We calculate the conditional expected value of both sides, which – along with $\Im_t \equiv E_t (Q_{t,t+1})$ – leads to the Euler equation for consumption:

$$\Im_t = \beta E_t \left[ \left( \frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right]$$ (32)

Log-linearization of (32) around the steady state allows to write the following dependence:

$$c_t - hc_{t-1} = E_t (c_{t+1} - hc_t) - \frac{1 - h}{\sigma} [i_t - (E_t p_{t+1} - p_t) + \ln \Im_t]$$ (33)

where lowercase variables are percentage deviations from the steady state for their uppercase counterparts. After basic simplifications, we obtain (see Smets and Wouters, 2003):

$$c_t = \frac{h}{1 + h} c_{t-1} + \frac{1}{1 + h} E_t c_{t+1} - \frac{1 - h}{(1 + h) \sigma} (i_t - E_t \pi_{t+1} - \rho)$$ (34)

where $i_t \equiv -\ln \Im_t$ denotes short-term nominal interest rate at $t$, $E_t \pi_{t+1} = E_t p_{t+1} - p_t$ – expected domestic consumer price growth, $\rho = -\ln \beta$ – natural interest rate corresponding to the households’ discount factor $\beta$ (p. Wicksell, 1907).

### 3.2 International prices

Define bilateral terms of trade between the home and foreign economy as:

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}}$$ (35)

Log-linearizing (35) around a symmetric steady state $S_t = 1$ – the law of one price in the tradable sector – leads to the following relationship:

$$s_t = p_{H,t} - p_{F,t}$$ (36)

Also, define internal terms of trade as price ratio between tradables and nontradables:

$$X_t \equiv \frac{P_{T,t}}{P_{N,t}}$$ (37)

An analogous approximation allows us to write:
\[ x_t = p_{T,t} - p_{N,t} \]  

(38)

When the steady state is asymmetric for some reason (e.g. persistent price level differentials due to heterogeneity in the level of economies’ development), then \( lnS_t \neq 0 \) and equation (36) takes the form:

\[ s_t = p_{H,t} - p_{F,t} - \bar{s} \]  

(39)

An asymmetric steady state has twofold implications. Firstly, equality (39) implies that a constant can appear in the equations containing \( s_t \). The value \( \bar{s} \) could probably be treated in an analogous manner to long-run equilibrium exchange rate in the sense of Williamson (1994), i.e. stabilizing the current account and production at their steady-state levels.

Secondly, generalizing the framework of Galí and Monacelli (2005) where a symmetric steady state was assumed, we obtain the following log-linearization of domestic CPI definition (see equation (23)):

\[
p_{T,t} = \frac{(1 - \alpha)S^{1-\eta}}{(1 - \alpha)S^{1-\eta} + \alpha}p_{H,t} + \frac{\alpha}{(1 - \alpha)S^{1-\eta} + \alpha}p_{F,t} \equiv (1 - \hat{\alpha})p_{H,t} + \hat{\alpha}p_{F,t} \]

(40)

Note that (40) simplifies to \( p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} \) when the steady state is symmetric. In addition, when the prices of domestic goods are lower than the foreign ones in the steady state \( \bar{S} < 1 \), then \( \alpha < \hat{\alpha} \) if \( \eta < 1 \), \( \alpha > \hat{\alpha} \) and \( \eta > 1 \) and like in the steady state if \( \eta = 1 \). Analogous conclusions apply to the log-linearization of (23).

Further derivations assume symmetric steady state for simplicity of exposition.

Using (39) and (21) we can write:

\[ p_{T,t} = p_{H,t} - \alpha s_t \]  

(41)

\[ p_t = p_{T,t} - \kappa x_t \]  

(42)

The real exchange rate \( Q_t \) (\( q_t \) for log-deviation from the steady state) versus the rest of the monetary union takes the form:

\[ q_t = p_t - p_t^* = (1 - \alpha - \alpha^*)s_t \]  

(43)

Real exchange rate appreciation is then linked to the appreciation of external terms of trade.

### 3.3 International risk sharing

Following Blessing (2008); Galí (2008); Kolasa (2008); Lipińska (2008) we assume that consumers can smooth consumption in international financial markets. With complete international markets for
securities, equality (44) is true also for any state $j$ (see Gali and Monacelli, 2005 for a more general version with flexible nominal exchange rate):

$$\frac{V_{t,t+1}}{P_t^i} (C^i_t - H^i_t)^{-\sigma} = \xi_{t,t+1} \beta \left( \frac{C^i_{t+1} - H^i_{t+1}}{P^i_{t+1}} \right)^{-\sigma} \frac{1}{P^i_{t+1}}$$  \hspace{1cm} (44)

The access to common capital market allows us to write an equation analogous to (45), derived from (44), with a common stochastic discount factor:

$$\beta \left( \frac{C^i_{t+1} - hC^i_t}{C^i_t - hC^i_{t-1}} \right)^{-\sigma} \left( \frac{P^i_t}{P^i_{t+1}} \right) = Q_{t,t+1}$$  \hspace{1cm} (45)

Combining (31) and (45), we obtain the following relationship:

$$(C_t - hC_{t-1})^{-\sigma} = \vartheta (C^*_t - h^*C^*_{t-1})^{-\sigma} Q_t$$  \hspace{1cm} (46)

Like Gali and Monacelli (2005) we assume $\vartheta = \beta = 1$ \hspace{.1cm} $\forall i$. The authors argue that it does not constrain the generality, affecting only initial conditions imposed on net foreign assets and states of nature. Log-linearizing (46) around the steady state, we derive the relationship between the domestic and foreign consumption and the real exchange rate (see Chari et al., 2002, for an empirical investigation):

$$\frac{\sigma}{1-h} (c_t - hc_{t-1}) = \frac{\sigma^*}{1-h^*} (c^*_t - h^*c^*_{t-1}) + q_t$$  \hspace{1cm} (47)

### 3.4 Producers

#### 3.4.1 Potential product and real marginal cost

The producers of variety $k$ in the tradable or nontradable bundle have the face the following production function (por. Gali, 2008):

$$Y^H_{t,k} = A^H_t N^H_{t,k}$$  \hspace{1cm} (48)

$$Y^N_{t,k} = A^N_t N^N_{t,k}$$  \hspace{1cm} (49)

whereby $\ln A^H_t = a^H_t = \rho^H a^H_{t-1} + \varepsilon_{a^H,t}$ and analogously for $N$. It implies a real marginal cost, common for producers in a given sector, calculated as:

$$mc^H_t = w_t - p_{H,t} - a^H_t$$  \hspace{1cm} (50)

$$mc^N_t = w_t - p_{N,t} - a^N_t$$  \hspace{1cm} (51)
Using log-linearized definitions of price aggregates (21) and (23) and the condition (18) we get:

\[ mc_t^H = -v + (w_t - p_t) + (p_t - p_{T,t}) + (p_{T,t} - p_{H,t}) - a_t^H = \sigma c_t + \phi n_t - \alpha s_t - \kappa x_t - a_t^H \]  
(52)

\[ mc_t^N = -v + (w_t - p_t) + (p_t - p_{T,t}) + (p_{T,t} - p_{N,t}) - a_t^N = \sigma c_t + \phi n_t - \alpha s_t - (1 - \kappa) x_t - a_t^N \]  
(53)

Aggregating (48) and (49) over the good types \( k \) and assuming identical technologies across all producers, we obtain sectoral production functions, implying the following relationship:

\[ n_t = n_t^N + n_t^H = y_t^N - a_t^N + y_t^H - a_t^H = y_t - a_t^N - a_t^H \]

Substituting this into (54) and (55) yields:

\[ mc_t^H = \sigma c_t + \phi y_t - \alpha s_t - \kappa x_t - (1 + \phi) a_t^H - \phi a_t^N \]  
(54)

\[ mc_t^N = \sigma c_t + \phi y_t - (1 - \kappa) x_t - \phi a_t^H - (1 + \phi) a_t^N \]  
(55)

Using (81), we can solve out \( c_t \), writing:

\[ mc_t^H = \left( \sigma \Gamma_c^{-1} + \phi \right) y_t - \left( \frac{1 - \mu}{w} \right) \sigma \Gamma_c^{-1} \Gamma_y y_t^* - \left( \alpha - \left( \frac{1 - u}{w} \right) \sigma \Gamma_c^{-1} \Gamma_s \right) s_t + \]
\[ - \left( \kappa + \sigma \Gamma_c^{-1} \Gamma_N \right) x_t + \left( \frac{1 - \mu}{w} \right) \sigma \Gamma_c^{-1} \Gamma_N x_t^* - (1 + \phi) a_t^H - \phi a_t^N \]  
(56)

\[ mc_t^N = \left( \sigma \Gamma_c^{-1} + \phi \right) y_t - \left( \frac{1 - \mu}{w} \right) \sigma \Gamma_c^{-1} \Gamma_y y_t^* - \left( \alpha - \left( \frac{1 - u}{w} \right) \sigma \Gamma_c^{-1} \Gamma_s \right) s_t + \]
\[ - \left( 1 - \kappa + \sigma \Gamma_c^{-1} \Gamma_N \right) x_t + \left( \frac{1 - \mu}{w} \right) \sigma \Gamma_c^{-1} \Gamma_N x_t^* - \phi a_t^H - (1 + \phi) a_t^N \]  
(57)

If prices were perfectly flexible, the real marginal cost would satisfy \( mc_t^H = -\mu^H \) and \( mc_t^N = -\mu^N \) for each \( t \). Solving (56) and (57) for the domestic production, using the equation above, we calculate the potential product at home:

\[ \bar{y}_t = \frac{\mu}{\sigma \Gamma_c^{-1} + \phi} + \left( \frac{1 - u}{w} \right) \frac{\sigma \Gamma_c^{-1} \Gamma_y y_t^*}{\sigma \Gamma_c^{-1} + \phi} + \left( \frac{1 - w}{w} \right) \frac{u - \sigma \Gamma_c^{-1} \Gamma_s}{\sigma \Gamma_c^{-1} + \phi} s_t + \]
\[ + \left( \frac{\kappa + \sigma \Gamma_c^{-1} \Gamma_N}{\sigma \Gamma_c^{-1} + \phi} \right) x_t + \left( \frac{1 - w}{w} \right) \frac{\sigma \Gamma_c^{-1} \Gamma_N x_t^*}{\sigma \Gamma_c^{-1} + \phi} + \left( \frac{1 + \phi}{\sigma \Gamma_c^{-1} + \phi} \right) a_t^H + \left( \frac{-\phi}{\sigma \Gamma_c^{-1} + \phi} \right) a_t^N \]  
(58)

### 3.4.2 Pricing decisions

There are price rigidities in the economy, modelled following the New Keynesian approach in the literature – by means of the Calvo (1983) scheme. In a given period, a fraction \( \theta \) of producers are not allowed to reoptimize their prices in reaction to economic innovations and must sell at the price from
the previous period. The probability of being allowed to reoptimize the price is equal across producers: $1 - \theta$, independently on the time elapsed since the last price change.

Some of the producers allowed to change their price do not really reoptimize. Following Galí and Gertler (1999) we assume that the change in price is partly implemented as an indexation to past inflation. This mechanism leads to a hybrid Phillips curve (see Galí and Gertler (1999); Galí et al. (2001)), empirically outperforming the purely forward-looking specifications in terms of goodness-of-fit. Following Kolasa (2008), inflation is modelled separately in the tradable and nontradable sector.

As Galí and Gertler (1999) we assume that a fraction $\theta$ of producers are able to change their price in each sector, which implies the following dependence between the price levels at $t - 1$ and $t$:

$$
\bar{p}_t^H = \theta^H \bar{p}_{t-1}^H + (1 - \theta^H) \bar{p}_t^H \quad \bar{p}_t^N = \theta^N \bar{p}_{t-1}^N + (1 - \theta^N) \bar{p}_t^N
$$

(59)

where $\bar{p}_t^H$ and $\bar{p}_t^N$ denote the newly set prices at $t$. Among the producers who reoptimize prices there is a fraction of $1 - \omega$ producers reoptimizing in an anticipatory manner as in Calvo (1983). They maximize the discounted flow of future profits, using all information available at the time of decision and taking into account future constraints on reoptimizing decisions. The rest of producers ($\omega$) reset their prices, according to past price dynamics:

$$
\tilde{p}_t^H = (1 - \omega^H) p_{b,t}^H + \omega^H p_{f,t}^H 
\tilde{p}_t^N = (1 - \omega^N) p_{b,t}^N + \omega^N p_{f,t}^N
$$

(60)

Prices set by the latter group of producers are modelled, as in Galí and Gertler (1999), as reoptimized prices from the previous period, indexed to past inflation:

$$
p_{b,t}^H = \bar{p}_{t-1}^H + \pi_{t-1}^H \quad p_{b,t}^N = \bar{p}_{t-1}^N + \pi_{t-1}^N
$$

(61)

One can show (see Galí and Gertler, 1999; Galí et al., 2001; Galí, 2008 for details) that the reoptimized prices satisfy the following conditions:

$$
p_{f,t}^H = \mu^H + (1 - \beta \theta^H) \sum_{s=0}^{\infty} (\beta \theta^H)^s E_t (mc_{t+s}^H + \bar{p}_{H,t+k})
$$

(62)

$$
p_{f,t}^N = \mu^N + (1 - \beta \theta^N) \sum_{s=0}^{\infty} (\beta \theta^N)^s E_t (mc_{t+s}^N + \bar{p}_{N,t+k})
$$

(63)

where $\mu^T \equiv \ln \frac{\varepsilon^T}{\varepsilon_{t-1}}$ and $\mu^N \equiv \ln \frac{\varepsilon^N}{\varepsilon_{t-1}}$ are log-markups in the steady state (or markups in an economy without price rigidities), $mc_t$ - real marginal cost at $t$.

### 3.5 Monetary policy

The central bank’s monetary policy is described by a Taylor rule with smoothing, which is a commonly applied description in the literature (see i.a. Kolasa, 2008; Gradzewicz and Makarski, 2008) and em-
pirically tested for the euro area (Sauer and Sturm, 2003). The common nominal interest rate is set according to the equation:

\[ i_t = (1 - \rho) [r^* + \pi^* + \gamma_\pi (\tilde{\pi}_t - \pi^*) + \gamma_y \tilde{y}_t] + \rho i_{t-1} \quad (64) \]

where \( i_t \) - central bank policy rate at \( t \), \( \tilde{y}_t \) - the output gap of a currency union, \( \tilde{\pi}_t \) - inflation rate in a currency union, \( r^* \) - natural interest rate, \( \pi^* \) - inflation target of the common central bank, \( \rho \in (0; 1) \) - smoothing parameter, \( \gamma_\pi > 1 \), \( \gamma_y > 0 \) - parameters of central bank’s response to deviations of inflation and output gap. The condition \( \gamma_\pi > 1 \) is necessary to satisfy the Taylor rule (Taylor, 1993), leading to a determinate equilibrium.

Treating \( \pi \) as deviation of inflation from the target, we rewrite equation (65) as:

\[ i_t = (1 - \rho) (\gamma_\pi \tilde{\pi}_t + \gamma_y \tilde{y}_t) + \rho i_{t-1} \quad (65) \]

Inflation and output gap in the currency union are aggregated over member countries:

\[ \tilde{\pi}_t = \int_0^1 \pi^*_j \, dj \quad (66) \]

\[ \tilde{y}_t = \int_0^1 y^*_j \, dj \quad (67) \]

In practice, for a finite number of economies, country weights (vector \( w_{n \times 1} \)) reflect relative sizes of economies (in a 2-country case: \( w_i = 1 - w \)).

3.6 Dynamic representation in equilibrium

3.6.1 Clearing of goods markets

Equilibrium on the world markets of individual goods requires equal overall production and consumption of every good \( k \):

\[
Y_{t,k}^H = \int_0^w C_{H,t,k} \, dj + \int_0^1 C^*_H \, dj =
\]

\[
= wC_{H,t,k} + (1 - w) C^*_H
\]

\[
= (1 - \alpha) (1 - \kappa) \left( \frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} \left( \frac{P_{H,t}}{P_{H,T}} \right)^{-\eta} \left( \frac{P_{H,t}}{P_T} \right)^{-\gamma} C_t +
\]

\[
+ \frac{1 - w}{w\alpha^*} (1 - \kappa^*) \left( \frac{P_{H,t,k}}{P_{H,T}} \right)^{-\varepsilon_T} \left( \frac{P_{H,T}}{P_{H,T}} \right)^{-\eta} \left( \frac{P_{H,t}}{P_T} \right)^{-\gamma} C^*_t =
\]

\[
= \left( \frac{P_{H,t,k}}{P_{H,t}} \right)^{-\varepsilon_T} \left[ (1 - \alpha) (1 - \kappa) \left( \frac{P_{H,t}}{P_{H,T}} \right)^{-\eta} \left( \frac{P_{H,t}}{P_T} \right)^{-\gamma} C_t + \frac{1 - w}{w\alpha^*} (1 - \kappa^*) \left( \frac{P_{H,t,k}}{P_{H,T}} \right)^{-\eta} \left( \frac{P_{H,t}}{P_T} \right)^{-\gamma} C^*_t \right]
\]

Plugging the above expression into the definition of aggregate domestic product,
Using the definition of internal terms of trade, (37), we use:

\[ Y^H_t = \int_0^1 Y_{t,k} \, dk \]  

yields:

\[ Y^H_t = (1 - \alpha)(1 - \kappa) \left( \frac{P_{H,t}}{F_t} \right)^{-\eta} \left( \frac{P_{T,t}}{F_t} \right)^{-\delta} C_t + \frac{1-w}{w} \alpha^* (1 - \kappa^*) \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-\eta} \left( \frac{P^*_T}{F^*_t} \right)^{-\delta} C^*_t = \]

\[ S_t \alpha^* X_t^{-\kappa \delta} \left[ (1 - \alpha)(1 - \kappa) C_t + \frac{1-w}{w} \alpha^* (1 - \kappa^*) S_t^{\alpha \eta - \eta' (1 - \alpha)} X_t^{-\kappa \delta} \right] \]

Log-linearizing around the steady state leads to the following equation:

\[ y^H_t = (1 - \alpha)(1 - \kappa) c_t + \frac{1-w}{w} \alpha^* (1 - \kappa^*) c_t - \frac{1-w}{w} \left\{ \alpha^* \eta^* (1 - \alpha^*) (1 - \kappa^*) - \alpha \eta \left[ \alpha^* (1 - \kappa^*) - \frac{w}{w-1} \right] \right\} s_t + \]

\[ + \kappa \delta \left[ \frac{1-w}{w} \alpha^* (1 - \kappa^*) - 1 \right] x_t - \frac{1-w}{w} \alpha^* \kappa \delta^* (1 - \kappa^*) x_t^* \]  

An analogous dependence can be written for the foreign economy:

\[ y^F_t = (1 - \alpha^*)(1 - \kappa^*) c^*_t + \frac{w}{1-w} \alpha (1 - \kappa) c_t - \frac{w}{1-w} \left\{ \alpha^* \eta^* \left[ \alpha (1 - \kappa) - \frac{w-1}{w} \right] - \alpha \eta \left[ \alpha (1 - \kappa) (1 - \kappa) \right] \right\} s_t + \]

\[ + \kappa^* \delta^* \left[ \frac{w}{1-w} \alpha (1 - \kappa) - 1 \right] x_t^* - \frac{w}{1-w} \alpha \kappa^* \delta^* (1 - \kappa) x_t \]  

Market clearing conditions for the nontradable sector can be written using (13) as:

\[ Y_{N,t} = C_{N,t} = \kappa \left( \frac{P_{N,t}}{F_t} \right)^{-\delta} C_t \quad Y^*_t = C^*_t = \kappa^* \left( \frac{P_{N,t}^*}{F_t^*} \right)^{-\delta} C^*_t \]

Using the definition of internal terms of trade, (37), we use:

\[ Y_{N,t} = \kappa X_t^{(1-\kappa)\delta} C_t \quad Y^*_t = \kappa^* (X_t^*)^{(1-\kappa^*)\delta} C_t^* \]

Log-linearizing (74) around the steady state leads to the following equations:

\[ y^N_t = \kappa (1 - \kappa) \delta x_t + \kappa c_t \quad y^*_t = \kappa^* (1 - \kappa^*) \delta^* x_t^* + \kappa^* c^*_t \]

Summing (75) with (71) and (80) yields, respectively,

\[ y_t = \left[ (1 - \alpha)(1 - \kappa) + \kappa \right] c_t + \frac{1-w}{w} \alpha^* (1 - \kappa^*) c_t^* + \]

\[ - \frac{1-w}{w} \left\{ \alpha^* \eta^* (1 - \alpha^*) (1 - \kappa^*) - \alpha \eta \left[ \alpha^* (1 - \kappa^*) - \frac{w}{w-1} \right] \right\} s_t + \]

\[ + \kappa \delta \left[ (1 - \kappa) + \frac{1-w}{w} \alpha^* (1 - \kappa^*) - 1 \right] x_t - \frac{1-w}{w} \alpha^* \kappa^* \delta^* (1 - \kappa^*) x_t^* \]  

19
\[ y_t^* = [(1 - \alpha^*) (1 - \kappa^*) + \kappa^*] c_t^* + \frac{w}{1-w} \alpha (1 - \kappa) t^* + \frac{w}{1-w} \alpha (1 - \kappa) \left[ \alpha (1 - \kappa) - \frac{w-1}{w} - \alpha \eta (1 - \alpha) (1 - \kappa) \right] s_t + + \kappa^* \delta^* \left[ (1 - \kappa^*) + \frac{w}{1-w} \alpha (1 - \kappa) - 1 \right] x_t^* + \frac{w}{1-w} \alpha \kappa (1 - \kappa) x_t \] (77)

The weighted sum of both equations is an equilibrium condition for the world’s markets:

\[ w y_t + (1 - w) y_t^* = wc_t + (1 - w) c_t^* \] (78)

From equality (78) we calculate \( c_t \) and \( c_t^* \), substituting into the equations (76) and (80):

\[ y_t = \frac{(1 - \alpha) (1 - \kappa) + \kappa - \alpha^* (1 - \kappa^*)}{1 - \alpha^* (1 - \kappa^*)} c_t + \frac{1-w}{w} \alpha (1 - \kappa) \left[ \frac{(1 - \alpha^*) (1 - \kappa^*)}{1 - \alpha^* (1 - \kappa^*)} y_t^* + \frac{1-w}{w} \alpha (1 - \kappa) \right] y_t^* + + \frac{1-w}{w} \alpha^* (1 - \kappa^*) - 1} \right] x_t - \frac{1-w}{w} \alpha (1 - \kappa) \left[ \frac{(1 - \alpha^*) (1 - \kappa^*)}{1 - \alpha^* (1 - \kappa^*)} x_t^* + \right] \right) \] (79)

\[ y_t^* = \frac{(1 - \alpha^*) (1 - \kappa^*) + \kappa^* - \alpha (1 - \kappa)}{1 - \alpha (1 - \kappa)} c_t^* + \frac{w}{1-w} \alpha (1 - \kappa) \left[ \frac{(1 - \alpha) (1 - \kappa) + \kappa - \alpha^* (1 - \kappa^*)}{1 - \alpha^* (1 - \kappa^*)} y_t + \frac{w}{1-w} \alpha (1 - \kappa) \right] y_t + + \frac{w}{1-w} \alpha^* (1 - \kappa^*) - 1} \right] x_t^* + \frac{w}{1-w} \alpha (1 - \kappa) \left[ \frac{(1 - \alpha) (1 - \kappa) + \kappa - \alpha^* (1 - \kappa^*)}{1 - \alpha^* (1 - \kappa^*)} x_t^* + \right] \right) \] (80)

We solve (79) for consumption:

\[ c_t = \Gamma_c^{-1} y_t - \left( \frac{1-w}{w} \right) \Gamma_c^{-1} y_t^* + \left( \frac{1-w}{w} \right) \Gamma_c^{-1} \Gamma_N s_t - \Gamma_c^{-1} \Gamma_N x_t + \left( \frac{1-w}{w} \right) \Gamma_c^{-1} \Gamma_N x_t^* \] (81)

Linking (81) and (33) we obtain the following relationship:

\[ y_t = \frac{h}{1+h} y_{t-1} + \frac{1}{1+h} E_t y_{t+1} - \left( \frac{1-h}{1+h} \right) \Gamma_c \left( c_t - E_t \pi_{t+1} - \rho \right) - \left( \frac{1-w}{w} \right) \Gamma_s \left( s_t - \frac{h}{1+h} s_{t-1} - \frac{1}{1+h} E_t s_{t+1} \right) + + \left( \frac{1-w}{w} \right) \Gamma_y \left( y_t^* - \frac{h}{1+h} y_{t-1}^* - \frac{1}{1+h} E_t y_{t+1}^* \right) + + \Gamma_N \left( x_t - \frac{h}{1+h} x_{t-1} - \frac{1}{1+h} E_t x_{t+1} \right) + - \left( \frac{1-w}{w} \right) \Gamma_N \left( x_t^* - \frac{h}{1+h} x_{t-1}^* - \frac{1}{1+h} E_t x_{t+1}^* \right) \] (82)
3.6.2 IS curves

The output gap is defined as $\hat{y}_t = y_t - \bar{y}_t$. Using equations (58) and (87) we can also express it as:

$$\hat{y}_t = \frac{h}{1+h} \hat{y}_{t-1} + \frac{1}{1+h} E_t \hat{y}_{t+1} - \frac{(1-h)\Gamma}{(1+h)\sigma} (i_t - E_t \pi_{t+1} - \bar{\rho})$$

$$- \left( \frac{1-w}{w} \right) \left( \Gamma_s + \frac{w}{\sigma} \frac{\sigma \Gamma_t^{-1} \Gamma_s}{\sigma \Gamma_t^{-1} + \phi} \right) (s_t - \frac{h}{1+h} s_{t-1} - \frac{1}{1+h} E_t s_{t+1}) +$$

$$+ \left( \frac{1-w}{w} \right) \Gamma_y \left( 1 + \frac{\sigma \Gamma_t^{-1}}{\sigma \Gamma_t^{-1} + \phi} \right) \left( \hat{y}_t - \frac{h}{1+h} \hat{y}_{t-1} - \frac{1}{1+h} E_t \hat{y}_{t+1} \right) +$$

$$+ \left( \Gamma_N - \frac{k + \sigma \Gamma_t^{-1} \Gamma_N}{\sigma \Gamma_t^{-1} + \phi} \right) \left( x_t - \frac{h}{1+h} x_{t-1} - \frac{1}{1+h} E_t x_{t+1} \right) +$$

$$- \left( \frac{1-w}{w} \right) \Gamma_N x (1 + \frac{\sigma \Gamma_t^{-1}}{\sigma \Gamma_t^{-1} + \phi}) \left( x_t^* - \frac{h}{1+h} x_{t-1}^* - \frac{1}{1+h} E_t x_{t+1}^* \right)$$

whereby

$$\bar{\rho} = \rho + \frac{(1+h)\sigma}{(1-h)} \frac{(1+\phi)}{1} \frac{\sigma \Gamma_t^{-1} + \phi}{\sigma \Gamma_t^{-1} + \phi} \left( a_t^H - \frac{h}{1+h} a_{t-1}^H - \frac{1}{1+h} E_t a_{t+1}^H \right) +$$

$$+ \frac{(1+h)\sigma}{(1-h)} \frac{\phi}{\sigma \Gamma_t^{-1} + \phi} \left( a_t^N - \frac{h}{1+h} a_{t-1}^N - \frac{1}{1+h} E_t a_{t+1}^N \right)$$

denotes the natural interest rate in the domestic economy (analogously for the foreign one). Equation (83) is an open economy IS curve. In particular, the competitiveness channel is governed by the $s_t$ term.

3.6.3 Phillips curve

Combined relationships (59)-(63) lead to the following hybrid Phillips curves in both domestic sectors:

$$\pi_t^H = \frac{\omega^H}{\sigma^H + \omega^H (1-\sigma^H)} \pi_{t-1}^H + \frac{\beta^H}{\sigma^H + \omega^H (1-\sigma^H)} E_t \pi_{t+1}^H +$$

$$+ \frac{(1-\omega^H) (1-\beta^H)}{\sigma^H + \omega^H (1-\sigma^H) (1-\beta^H)} mc_t^H$$

$$\pi_t^N = \frac{\omega^N}{\sigma^N + \omega^N (1-\sigma^N)} \pi_{t-1}^N + \frac{\beta^N}{\sigma^N + \omega^N (1-\sigma^N) (1-\beta^N)} E_t \pi_{t+1}^N +$$

$$+ \frac{(1-\omega^N) (1-\beta^N)}{\sigma^N + \omega^N (1-\sigma^N) (1-\beta^N)} mc_t^N$$

where $mc_t^H$ now denotes the deviation of real marginal cost from its long-run value in the domestic tradable sector (analogously for nontradables). The real marginal costs in both sectors: $mc_t^H$ and $mc_t^N$ are determined by the equations (56) and (57).

3.6.4 Model equations

The dynamic model is composed of output equations (87), Phillips curves (85) and (86), real marginal cost (56) and (57) and consumption (81) along with their foreign counterparts, international risk sharing condition (47), equation of common monetary policy (65) and a set of identities defining the aggregate values for the monetary union (66) i (67), aggregate price dynamics and deflators. Model equations are listed in the Appendix.
4 Estimation

The presence of expectational components in the model requires using specific estimation techniques. As shown by Torój (2009b), careful estimation of forward- and backward-looking parameters in the IS and Phillips curves is critical for accurate modelling of adjustment dynamics after asymmetric shocks.

In the literature, there are two standard manners of handling this problem. Firstly, following a seminal paper of Galí and Gertler (1999), the system can be estimated equation-by-equation using generalized method of moments (see Hansen, 1982). Secondly, one can specify a closed system and solve out the forward-looking components using standard algorithms (Blanchard and Kahn, 1980; Klein, 2000; Sims, 2001) and estimate the system as structural VAR. It is commonly argued that the latter approach outperforms the former in terms of small sample bias, identification and instrument optimality (see i.a. Fuhrer and Rudebusch, 2004; Mavroeidis, 2005; Lindé, 2005). However, as both approaches are usually compared in the literature and the results for Poland and Slovakia require some robustness check, we apply both techniques.

4.1 Single equation analysis (GMM)

Building upon (82), we estimate the following equation for Poland and Slovakia:

\[ y_t = c + (1 - \gamma) y_{t-1} + \gamma E_t y_{t+1} - \phi (i_t - E_t \pi_{t+1}) - \varphi (s_t - (1 - \gamma) s_{t-1} - \gamma E_t s_{t+1}) + \rho \left( y_{t*} - (1 - \gamma) y_{t*-1} - \gamma E_t y_{t*+1}^* \right) \]  

A similar estimation setup for a closed, one-sector economy was applied by Fuhrer and Rudebusch (2004). In line with the standard approach in the literature, the instrument set contains lags of all explanatory variables (Fuhrer and Rudebusch, 2004; Goodhart and Hofmann, 2005). Note that, to hold the results comparable for both countries in question, the sample and instrument set is the same for both economies (see Table 1).

4.2 System analysis (FIML)

An alternative estimation strategy takes into account full model specification. The expectational terms in the IS equation are endogenous elsewhere in the model. This is why they can be solved for, given the rationality of agents’ expectations.

The log-linearized model can be summarized in a matrix form as:

\[ A E_t x_{t+1} = B x_t + C \epsilon_t \]
Table 1: IS curve estimates for Poland and Slovakia (GMM)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Poland</th>
<th></th>
<th>Slovakia</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>p-value</td>
<td>Estimate</td>
<td>p-value</td>
</tr>
<tr>
<td>c</td>
<td>-0.031</td>
<td>0.326</td>
<td>0.011</td>
<td>0.887</td>
</tr>
<tr>
<td>γ</td>
<td>0.739</td>
<td>0.000</td>
<td>0.525</td>
<td>0.000</td>
</tr>
<tr>
<td>φ</td>
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<td>0.095</td>
<td>0.034</td>
<td>0.441</td>
</tr>
<tr>
<td>ϕ</td>
<td>0.103</td>
<td>0.004</td>
<td>0.323</td>
<td>0.030</td>
</tr>
<tr>
<td>ρ</td>
<td>0.508</td>
<td>0.025</td>
<td>5.353</td>
<td>0.002</td>
</tr>
<tr>
<td>MCI-ratio</td>
<td>φ/φ (90% conf. int.)</td>
<td>0.559 (-0.135;1.233)</td>
<td>0.106 (-0.087;0.298)</td>
<td></td>
</tr>
</tbody>
</table>

Sample: 1999q1-2009q2. Instruments: domestic output gap (lags 1 to 3), changes in euro area output gap (lags 1 to 3), log-differences in real exchange rate vis-a-vis the euro area (lags 1 to 3), inflation rate (lag 1).

After solution, the model is

\[ x_t = M x_{t-1} + N \varepsilon_t \]  \hspace{1cm} (89)

where \( M_{n_1 \times n_1}(A, B, C) \) and \( N_{n_1 \times k}(A, B, C) \), \( n_1 \) – number of variables in the reduced system, \( k \) – number of structural shocks in the model.

In rational expectations models, it is common to assume AR(1) residuals by construction (see e.g. Mavroeidis, 2005, for motivation):

\[ \varepsilon_t = \Phi \varepsilon_{t-1} + v_t \]  \hspace{1cm} (90)

where \( \Phi \) has nonexplosive eigenvalues and \( \varepsilon \sim N(0, D) \).

TO BE COMPLETED

5 How the MCI-ratio matters in adjustment dynamics under EMU: simulation analysis

TO BE COMPLETED

6 MCI-ratio and revaluations in ERM II: lessons from the Slovak experience

TO BE COMPLETED
7 Conclusion

TO BE COMPLETED

References


24


Wald A. (1943): Tests of Statistical Hypotheses Concerning Several Parameters When the Number of Observations Is Large, Transactions of the American Mathematical Society, 54 (3), 426–82.


Appendix: Log-linearized model

\[ y_t = \frac{h}{h+h} y_{t-1} + \frac{1}{h+h} E_t y_{t+1} - \frac{(1-h) \Gamma_x}{(1+h)\sigma} (i_t - E_t \pi_{t+1} - \rho) + \]
\[ - \frac{(1-w)}{w} \Gamma_s \left( s_t - \frac{h}{h+h} s_{t-1} - \frac{1}{h+h} E_t s_{t+1} \right) + \]
\[ + \left( \frac{w}{1-w} \right) \Gamma_y \left( y_t^* - \frac{h}{h+h} y_{t-1} - \frac{1}{h+h} E_t y_{t+1}^* \right) + \]
\[ + \Gamma_N \left( x_t - \frac{h}{h+h} x_{t-1} - \frac{1}{h+h} E_t x_{t+1} \right) \]
\[ - \frac{(1-w)}{w} \Gamma_N \left( x_t^* - \frac{h}{h+h} x_{t-1}^* - \frac{1}{h+h} E_t x_{t+1}^* \right) \]

\[ y_t^* = \frac{h^*}{1+h^*} y_{t-1} + \frac{1}{1+h^*} E_t y_{t+1} - \frac{(1-h^*) \Gamma_x^*}{1+h^*} (i_t - E_t \pi_{t+1} - \rho^*) + \]
\[ + \left( \frac{w}{1-w} \right) \Gamma_s^* \left( s_t - \frac{h^*}{h^*+h^*} s_{t-1} - \frac{1}{h^*+h^*} E_t s_{t+1} \right) + \]
\[ + \left( \frac{w}{1-w} \right) \Gamma_y^* \left( y_t^* - \frac{h^*}{h^*+h^*} y_{t-1} - \frac{1}{h^*+h^*} E_t y_{t+1}^* \right) + \]
\[ + \Gamma_N^* \left( x_t^* - \frac{h^*}{h^*+h^*} x_{t-1}^* - \frac{1}{h^*+h^*} E_t x_{t+1}^* \right) \]

\[ \pi_{H,t} = \frac{\sigma_{\pi+\omega H}^{\omega \rho}}{\sigma_{\pi+\omega H}^{\omega \rho}} \pi_{H,t-1} + \frac{\sigma_{\pi+\omega H}^{\omega \rho}}{\sigma_{\pi+\omega H}^{\omega \rho}} E_t \pi_{H,t+1} + \]
\[ + \frac{\sigma_{\pi+\omega H}^{\omega \rho}}{\sigma_{\pi+\omega H}^{\omega \rho}} \left( 1 - \frac{\sigma_{\pi+\omega H}^{\omega \rho}}{\sigma_{\pi+\omega H}^{\omega \rho}} \right) \frac{\sigma_{\pi+\omega H}^{\omega \rho}}{\sigma_{\pi+\omega H}^{\omega \rho}} \frac{E_t \pi_{H,t+1}}{E_t \pi_{H,t+1}} \frac{m_{c_{H}}^{H}}{m_{c_{H}}^{H}} \]

\[ \pi_{N,t} = \frac{\sigma_{\pi+\omega N}^{\omega \rho N}}{\sigma_{\pi+\omega N}^{\omega \rho N}} \pi_{N,t-1} + \frac{\sigma_{\pi+\omega N}^{\omega \rho N}}{\sigma_{\pi+\omega N}^{\omega \rho N}} E_t \pi_{N,t+1} + \]
\[ + \frac{\sigma_{\pi+\omega N}^{\omega \rho N}}{\sigma_{\pi+\omega N}^{\omega \rho N}} \left( 1 - \frac{\sigma_{\pi+\omega N}^{\omega \rho N}}{\sigma_{\pi+\omega N}^{\omega \rho N}} \right) \frac{\sigma_{\pi+\omega N}^{\omega \rho N}}{\sigma_{\pi+\omega N}^{\omega \rho N}} \frac{E_t \pi_{N,t+1}}{E_t \pi_{N,t+1}} \frac{m_{c_{N}}^{H}}{m_{c_{N}}^{H}} \]

\[ \pi_{F,t} = \frac{\sigma_{\pi+\omega F}^{\omega \rho F}}{\sigma_{\pi+\omega F}^{\omega \rho F}} \pi_{F,t-1} + \frac{\sigma_{\pi+\omega F}^{\omega \rho F}}{\sigma_{\pi+\omega F}^{\omega \rho F}} E_t \pi_{F,t+1} + \]
\[ + \frac{\sigma_{\pi+\omega F}^{\omega \rho F}}{\sigma_{\pi+\omega F}^{\omega \rho F}} \left( 1 - \frac{\sigma_{\pi+\omega F}^{\omega \rho F}}{\sigma_{\pi+\omega F}^{\omega \rho F}} \right) \frac{\sigma_{\pi+\omega F}^{\omega \rho F}}{\sigma_{\pi+\omega F}^{\omega \rho F}} \frac{E_t \pi_{F,t+1}}{E_t \pi_{F,t+1}} \frac{m_{c_{T}}^{H}}{m_{c_{T}}^{H}} \]

\[ m_{c_{H}}^{H} = (\sigma_{\Gamma_{c}^{1}} + \phi) y_t - \left( \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} x_t^* + \left[ \alpha - \left( \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* \right] s_t + \]
\[ - \left( \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* - \left( \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* - \left( 1 - \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* \]

\[ m_{c_{N}}^{N} = (\sigma_{\Gamma_{c}^{1}} + \phi) y_t - \left( \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} x_t^* + \left[ \alpha - \left( \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* \right] s_t + \]
\[ - \left( 1 - \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* - \left( 1 - \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* - \left( \frac{1-w}{w} \right) \sigma_{\Gamma_{c}^{1}} \Gamma_N x_t^* \]
\[ c_t = \Gamma_c^{-1} y_t - \left(\frac{1-w}{w}\right) \Gamma_c^{-1} \Gamma_s y_t^* + \left(\frac{1-w}{w}\right) \Gamma_c^{-1} \Gamma_N x_t + \left(\frac{1-w}{w}\right) \Gamma_c^{-1} \Gamma_N x_t^* \]

\[ c_t^* = \Gamma_c^{-1} y_t^* - \left(\frac{w}{1-w}\right) \Gamma_c^{-1} \Gamma_s y_t - \left(\frac{w}{1-w}\right) \Gamma_c^{-1} \Gamma_s y_t^* - \left(\frac{w}{1-w}\right) \Gamma_c^{-1} \Gamma_N x_t + \left(\frac{w}{1-w}\right) \Gamma_c^{-1} \Gamma_N x_t^* \]

\[ i_t = (1-\rho) [r^* + \pi_t + \gamma_t (\tilde{\pi}_t - \pi^*) + \gamma_y \tilde{y}_t] + \rho i_{t-1} \]

\[ \tilde{\pi}_t = \int_0^1 \pi_t^j \, dj \]

\[ \tilde{y}_t = \int_0^1 y_t^j \, dj \]

\[ \frac{\sigma}{1-h_c} (c_t - h_c i_{t-1}) = \frac{\sigma^*}{1-h_c^*} (c_t^* - h_c^* i_{t-1}) + (1-\alpha - \alpha^*) \pi_t \]

\[ \pi_t^T = (1-\alpha) \pi_{H,t} + \alpha \pi_{F,t} \]

\[ \pi_t^{T^*} = (1-\alpha^*) \pi_{F,t} + \alpha^* \pi_{H,t} \]

\[ \pi_t = (1-\kappa) \pi_{T,t} + \kappa \pi_{N,t} \]

\[ \pi_t^* = (1-\kappa^*) \pi_{T,t}^* + \kappa^* \pi_{N,t}^* \]

\[ s_t = p_{H,t} - p_{F,t} \]

\[ x_t = p_{T,t} - p_{N,t} \]

\[ x_t^* = p_{T,t}^* - p_{N,t}^* \]

\[ p_{H,t} = p_{H,t-1} + \pi_{H,t} \]

\[ p_{F,t} = p_{F,t-1} + \pi_{F,t} \]
\[ p_{N,t} = p_{N,t-1} + \pi_{N,t} \]

\[ p_{N,t}^* = p_{N,t-1}^* + \pi_{N,t}^* \]

\[ p_{T,t} = p_{T,t-1} + \pi_{T,t} \]

\[ p_{T,t}^* = p_{T,t-1}^* + \pi_{T,t}^* \]