A metodology for assessment of metamodel quality

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Abstract

Consider a mathematical model S of some macro- or microeconomic problem that is stochastic and complex enough that it is not possible to solve it analytically and one has to use simulation to analyze its properties. In general S is a mapping from *simulation parameter* space $\mathbb{D}$ into a space of random variables $\mathbb{Y}$ called *simulation outputs*.

Assume that the researcher is interested in expected value of the simulation output, i.e. $R(d) = E(S(d))$ and that this expected value exists. Function $R\colon \mathbb{D} \to \mathbb{R}$ is often called *response surface* in the literature. A *simulation metamodel* is a mapping $M\colon \mathbb{D} \to \mathbb{R}$ that whose goal is to approximate R. Usually functional form of response surface R is unknown and metamodel M is selected (estimated) from a given family of functions $\mathcal{F}$.

In this text we assume that a metamodel M was estimated by the researcher and now she asks the question how well M approximates R. In order to preserve generality the presented analysis will be independent from functional form of M. Standard measures of fit of a metamodel M to underlying data (like R^2 or MSE) measure distance of $M(d)$ form sample of S over a sample from parameter space D. We claim that actually researcher is interested in measurement of $M(d) - R(d)$ (i.e. model bias) given some assumption about distribution of d in parameter space D. To understand why observe that for given $d \in \mathbb{D}$:

$$D^2(M(d) - S(d)) = (M(d) - R(d))^2 + D^2(S(d)).$$

We can see that $M(d) - S(d)$ mixes two kinds of error: metamodel bias $B(d) = M(d) - R(d)$ and simulation output variance $D^2(S(d))$ (which in general can be heterogeneous). Therefore high value of $D^2(M(d) - S(d))$ may mean that we have a bad metamodel, that simulation output is highly uncertain or both. Therefore, formally, we want to test:

$$H_0\colon \sup_{d\in\mathbb{D}} |B(d)| = 0 \quad \text{against} \quad H_1\colon \sup_{d\in\mathbb{D}} |B(d)| > 0.$$

In the text we discuss tests of this hypothesis existing in the literature and propose several new tests. Their performance is compared on example of sample simulation models of economic problems.